

1. When we say a problem is decidable? Give an example of undecidable problem?

A problem whose language is recursive is said to be decidable. Otherwise the problem is said to be undecidable. Decidable problems have an

algorithm that takes as input an instance of the problem and determines whether the answer to that instance is “yes” or “no”.

(eg) of undecidable problems are (1) Halting problem of the TM.

2. Give examples of decidable problems.

1. Given a DFMSM M and string w , does M accept w ?
2. Given a DFMSM M is $L(M) = \Phi$?
3. Given two DFMSMs M_1 and M_2 is $L(M_1) = L(M_2)$?
4. Given a regular expression α and a string w , does α generate w ?
5. Given a NFSM M and string w , does M accept w ?

3. Give examples of recursive languages?

- i. The language L defined as $L = \{ \langle M \rangle, w : M \text{ is a DFMSM that accepts } w \}$ is recursive.
- ii. L defined as $\{ \langle M_1 \rangle \cup \langle M_2 \rangle : \text{DFMSMs } M_1 \text{ and } M_2 \text{ and } L(M_1) = L(M_2) \}$ is recursive.

4. Differentiate recursive and recursively enumerable languages.

Recursive languages

1. A language is said to be recursive if and only if there exists a membership algorithm for it.
2. A language L is recursive iff there is a TM that decides L . (Turing decidable languages). TMs that decide languages are algorithms.

Recursively enumerable languages

1. A language is said to be r.e if there exists a TM that accepts it.
2. L is recursively enumerable iff there is a TM that semi-decides L . (Turing acceptable languages). TMs that semi-decides languages are not algorithms

5. What are UTMs or Universal Turing machines?

Universal TMs are TMs that can be programmed to solve any problem, that can be solved by any Turing machine. A specific Universal Turing machine U is:

Input to U: The encoding “M” of a TM M and encoding “w” of a string w. Behavior : U halts on input “M” “w” if and only if M halts on input w.

6. What are the crucial assumptions for encoding a TM?

There are no transitions from any of the halt states of any given TM. Apart from the halt state, a given TM is total.

7. What properties of recursive enumerable sets are not decidable?

Emptiness

Finiteness

Regularity

Context-freeness.

8. Define L_ℓ . When is ℓ a trivial property?

L_ℓ is defined as the set $\{ \langle M \rangle \mid L(M) \text{ is in } \ell. \}$

ℓ is a trivial property if ℓ is empty or it consists of all r.e languages.

9. What is a universal language L_u ?

The universal language consists of a set of binary strings in the form of pairs (M,w) where M is TM encoded in binary and w is the binary input string.

$L_u = \{ \langle M,w \rangle \mid M \text{ accepts } w \}$.

10. What is a Diagonalization language L_d ?

The diagonalization language consists of all strings w such that the TM M whose code is w does not accept when w is given as input.

11. What properties of r.e sets are recursively enumerable?

$L \neq \Phi$

L contains at least 10 members.

w is in L for some fixed w. $L \cap Lu \neq \Phi$

12. What properties of r.e sets are not r.e?

$L = \Phi$

$L = \Sigma^*$.

L is recursive

L is not recursive. L is singleton.

L is a regular set. $L - Lu \neq \Phi$

13. What is canonical ordering?

Let Σ^* be an input set. The canonical order for Σ^* as follows. List words in order of size, with words of the same size in numerical order. That is let $\Sigma = \{$

$x_0, x_1, \dots, x_{t-1} \}$ and x_i is the digit i in base t .

(e.g) If $\Sigma = \{ a, b \}$ the canonical order is $\epsilon, a, b, aa, ab, \dots$

14. How can a TM acts as a generating device?

In a multi-tape TM, one tape acts as an output tape, on which a symbol, once written can never be changed and whose tape head never moves left. On that output

tape, M writes strings over some alphabet Σ , separated by a marker symbol #, $G(M)$ (where $G(M)$ is the set w in Σ^* such that w is finally printed between a pair of #'s on the output device).

15. What are the different types of grammars/languages?

- Unrestricted or Phase structure grammar.(Type 0 grammar).(for TMs)
- Context sensitive grammar or context dependent grammar (Type1)(for

Linear Bounded Automata)

- Context free grammar (Type 2) (for PDA)

• Regular grammar (Type 3) (for Finite Automata). This hierarchy is called as Chomsky Hierarchy.

16. What is a PS or Unrestricted grammar?

A grammar without restrictions is a PS grammar. Defined as $G=(V,T,P,S)$ With P as :

$\Phi A \psi \rightarrow \Phi \alpha \psi$ where A is variable and $\Phi \alpha \psi$ is replacement string. The languages generated by unrestricted grammars are precisely those accepted by Turing machines.

17. State a single tape TM started on blank tape scans any cell four or more times is decidable?

If the TM never scans any cell four or more times , then every crossing sequence is of length at most three. There is a finite number of distinct crossing

sequence of length 3 or less. Thus either TM stays within a fixed bounded number of tape cells or some crossing sequence repeats.

18.Does the problem of “ Given a TM M ,does M make more than 50 moves on input B “?

Given a TM M means given enough information to trace the processing of a fixed string for a certain fixed number of moves. So the given problem is

decidable.

19. Show that AMBIGUITY problem is un-decidable.

Consider the ambiguity problem for CFGs. Use the “yes-no” version of AMB.

An algorithm for FIND is used to solve AMB. FIND requires producing a word with

two or more parses if one exists and answers “no” otherwise. By the reduction of

AMB to FIND we conclude there is no algorithm for FIND and hence no algorithm for AMB.

20.State the halting problem of TMs.

The halting problem for TMs is:

Given any TM M and an input string w, does M halt on w?

This problem is undecidable as there is no algorithm to solve this problem.

21.Define PCP or Post Correspondence Problem.

An instance of PCP consists of two lists , $A = w_1, w_2, \dots, w_k$

and $B = x_1, \dots, x_k$ of strings over some alphabet Σ .This instance of PCP has a

solution if there is any sequence of integers i_1, i_2, \dots, i_m with $m \geq 1$ such that

$$w_{i_1}, w_{i_2}, \dots, w_{i_m} = x_{i_1} x_{i_2} \dots x_{i_m}$$

The sequence i_1, i_2, \dots, i_m is a solution to this instance of PCP.

22. Define MPCP or Modified PCP.

The MPCP is : Given lists A and B of K strings from Σ^* , say

$$A = w_1, w_2, \dots, w_k \text{ and } B = x_1, x_2, \dots, x_k$$

does there exist a sequence of integers i_1, i_2, \dots, i_r such that $w_{i_1} w_{i_2} \dots w_{i_r} = x_{i_1} x_{i_2} \dots x_{i_r}$?

23. What is the difference between PCP and MPCP?

The difference between MPCP and PCP is that in the MPCP, a solution is required to start with the first string on each list.

24. What are the concepts used in UTMs?

Stored program computers.

Interpretive Implementation of Programming languages.

Computability.

25. What are (a) recursively enumerable languages (b) recursive sets?

The languages that are accepted by TM are said to be recursively enumerable (r. e.)

languages. Enumerable means that the strings in the language can be enumerated by

the TM. The class of r. e. languages include CFL's.

The recursive sets include languages accepted by at least one TM that halts on all inputs.

26. When a recursively enumerable language is said to be recursive? Is it true that the language accepted by a non-deterministic Turing machine is different from recursively enumerable language?

A language L is recursively enumerable if there is a TM that accepts L and recursive if there is a TM that recognizes L. Thus r.e language is Turing acceptable and

recursive language is Turing decidable languages.

No , the language accepted by non-deterministic Turing machine is same as recursively enumerable language.

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